4 Image Restoration

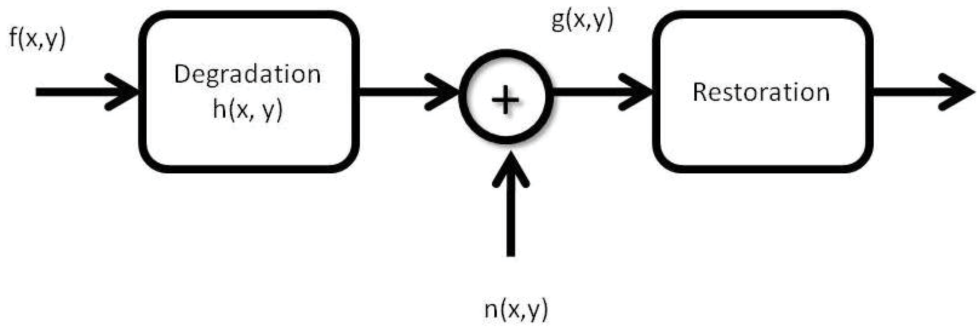
4.1 Image degradation and restoration

A degraded image is a clean image with an additive noise term. In a mathematical language, given a clean image f(x, y) and a noise function n(x, y), the degraded image is generated as follows:

$$g(x,y) = h(x,y) * f(x,y) + n(x,y) \tag{4.1.1}$$

where $h(x, y)$ is the transformation function from the clean image to the degraded image in spatial domain and "*" is convolution.

Restoration is the converse process of degradation. That is to say, the purpose of image restoration is to seek a maximum similarity of the original clean image after an appropriate process is applied. Such a degradation/restoration procedure can be represented in the following graph:



It is essential to understand how noise affects images before an optimal restoration method can be introduced. Without this knowledge, it will not be possible to discover a proper solution. In the nest section, the noise models will be introduced, where up to date research outcomes are presented.

4.2 Noise analysis

Image noise is normally due to environmental conditions (e.g. light, temperature, etc), sensor quality and human interference. Of these factors, the first two seems to be dominant in real practice. For example, a camera of low resolution easily leads to fuzzy imaging outcomes. A moving camera can collect image sequences with strong motion blurred effects.

To perform appropriate noise analysis one of the possible ways is to understand the noise models behind the image signals. After these noise models have been identified, it may become much easier to apply corresponding strategies for problem solution. In general, these noise models can be grouped into time and frequency domains, the latter of which considers the difference between the frequency properties of the clean image and noise. More details can be found in the following.

The first noise model is **Gaussian**. This is a well recognised noise model due to its mathematical tractability and popularity of use. To better understand the principle of Gaussian models, one of the options is to extract its probability distribution function (PDF). The PDF of Gaussian models is expressed by

$$p(s) = \frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{(s-\mu)^2}{2\sigma^2}} \tag{4.2.1}$$

where s is gray level, μ is the mean of s and σ is its variance. The difference between the image signals and the average value is anti-proportional to their probability values.

The second one is **uniform noise model**. The PDF of uniform noise is

$$p(s) = \begin{cases} \dfrac{1}{a-b} \\ 0 \end{cases} \tag{4.2.2}$$

where the upper case results from $a \geq s \geq b$; otherwise the lower case stands. This indicates that the probability values will be the same no matter where each pixel locates.

The third one is **Rayleigh noise**, whose PDF is delineated by

$$p(s) = \begin{cases} \frac{2}{b}(s-a)\exp^{-\frac{(s-a)^2}{b}} \\ 0 \end{cases} \qquad b(4.2.3)$$

where the upper case results from $s \geq a$ (a and b are two constants); otherwise the lower one is valid. This noise model results a deformed shape of thr Gaussian model.

Another quite common noise model is **salt and pepper noise** which has a PDF as follows

$$p(s) = \begin{cases} P_a \\ P_b \\ 0 \end{cases} \qquad (4.2.4)$$

where the upper case is due to $s = a$, the middle case is due to $s = b$ and otherwise the lower case is valid.

Figure 40 illustrates some images with additive noise (the noise power is also included in the caption).

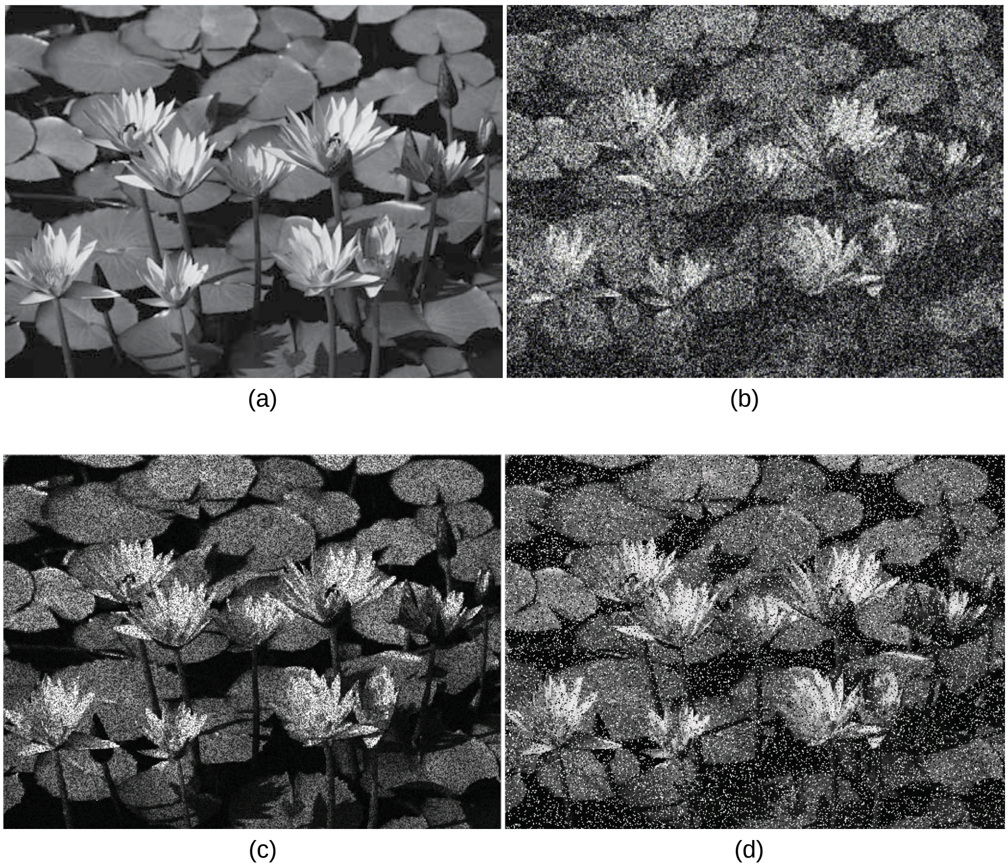


Figure 40 Noisy images with different additive noise: (a) original image, (b) Gaussian (0, 0.2) (mean, variance), (c) speckle (0.2) and (d) salt & pepper (0.2).

4.3 Restoration with spatial analysis

A number of approaches have been established to handle the image restoration problem. These normally consist of spatial and frequency based approaches. In this subsection only spatial analysis based methods are concerned. Of the developed schemes, spatial filtering is one of the frequently investigated areas and its applications have been well developed. Some examples are summarized here.

Geometric mean filter: This is a model that incorporates geometric mean. Each restored image pixel is the mean value of the pixels falling in the investigated area. The underlying mathematical expression for the restoration is followed:

$$\hat{f}(x,y) = (\prod_{(q,t)\in Q} \tilde{f}(q,t)^{\frac{1}{m}} \tag{4.3.1}$$

where (m, n) indicates the size of a subimage window. Examples of applying this filter to the noisy images (shown in **Figure 40**) are given in **Figure 41**. It demonstrates that this filter is more suitable to handle the case of speckle noise.

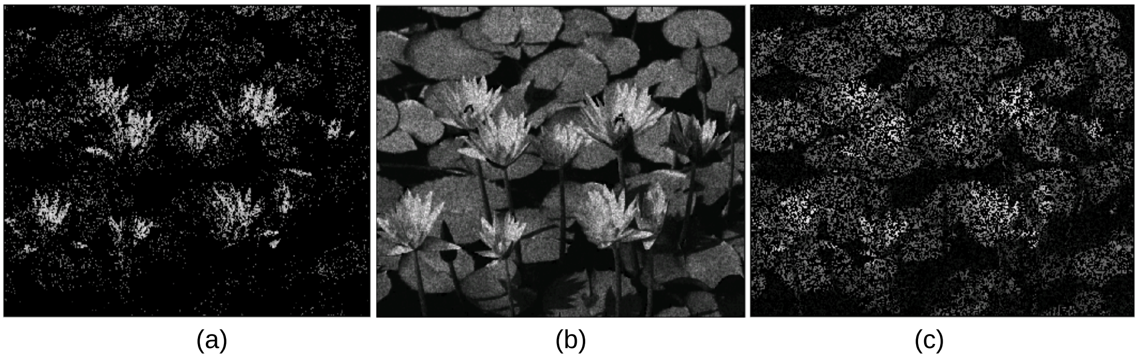
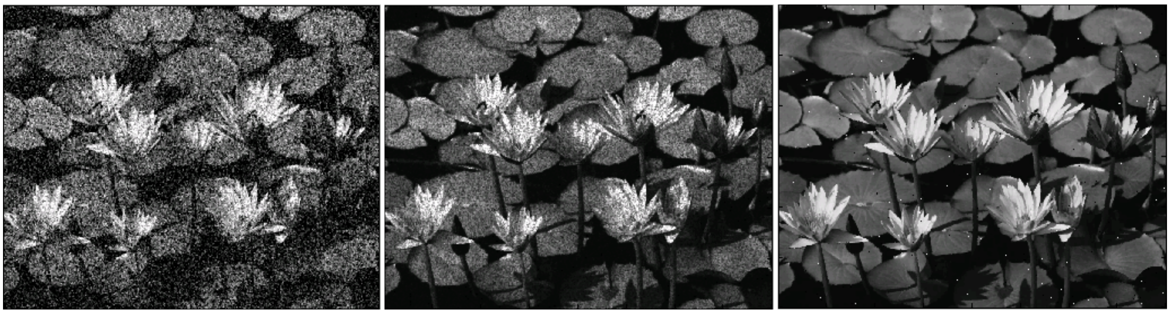


Figure 41 Examples of using the geometric mean filter, where (a) Gaussian, (b) speckle and (c) salt & pepper.

Median filter: This is a statistics based filtering algorithm. Any image pixel can be replaced by the median of the image pixels in the neighborhood of that pixel:

$$\hat{f}(x,y) = median\{\tilde{f}(q,t)\} \tag{4.3.2}$$

Figure 42 illustrates the outcomes of using the median filter to the images shown in **Figure 40**. It is clear that this median filter has the best performance when we restore the salt & pepper image.



(a) (b) (c)

Figure 42 Outcomes of applying the median filter to the noisy images.

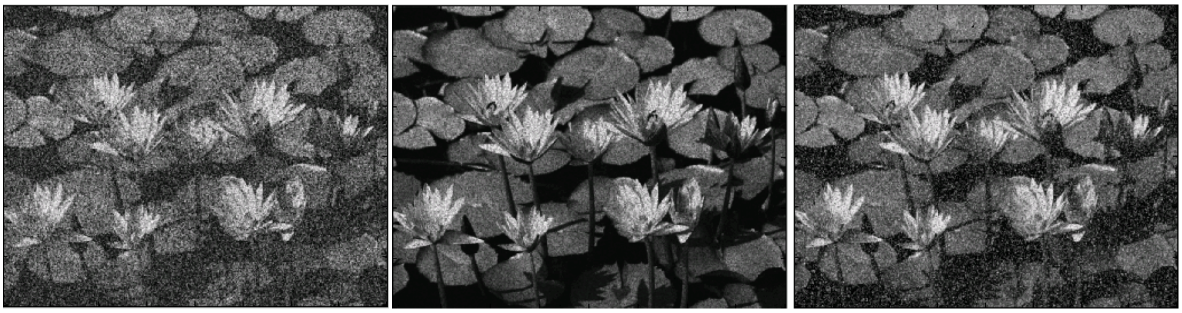
Max and min filters: The median filter takes the mid point of the image intensity. On contrary, max and min filters are designed to extract the maximum or minimum point in the ranked numbers. This filter effectively works in some extreme cases, where the noise makes the image intensities outstanding. However, it will not perform well in the presence of regular noise distributions.

Adaptive filters: These filters are intended to adapt their behaviors depending on the characteristics of the image area to be filtered. This is based on a simple fact that many signals actually are the combination of the known or anonymous noise resources and hence it is unlikely to obtain an appropriate noise model to describe the characteristics of the noise.

One commonly used adaptive filters is local noise reduction filter, where the denoising result can be represented by

$$\hat{f}(x,y) = \tilde{f}(x,y) - \frac{\sigma_1^2}{\sigma_2^2}[\tilde{f}(x,y) - m] \tag{4.3.3}$$

where σ_1^2 is estimated variance of the noise corrupting the image, m and σ_2^2 are the mean and variance of the pixels in the sub-image. These variables sometimes can only be available through historic estimation. The performance of this filter is revealed in **Figure 43**.



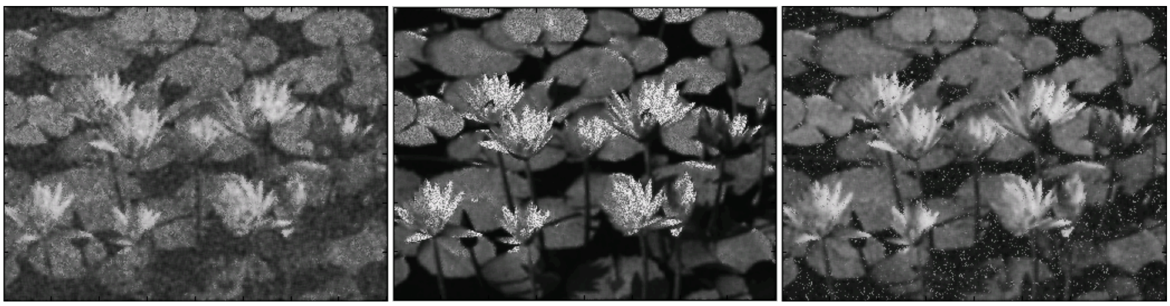
(a) (b) (c)

Figure 43 Illustration of the adaptive filter in image denoising.

Wiener filter: This is a statistical approach to pursue an outcome that can minimize the mean square error between the restored and original images, given that the image and noise are uncorrelated with normal distribution:

$$e^2 = E[f(x,y) - \hat{f}(x,y]^2 \tag{4.3.4}$$

This leads to the outcomes shown in **Figure 44**. This shows that the restored images of speckle and salt & pepper noise are quite similar.



(a) (b) (c)

Figure 44 Performance of the Wiener filter in image restoration.

4.4 Restoration with frequency analysis

In the frequency domain the restored image can be represented as follows:

$$G(u,v) = H(u,v) * F(u,v) + N(u,v) \tag{4.4.1}$$

Where *G*, *H*, *F* and *N* are the Fourier transforms of *g*, *h*, *f* and *n*, respectively. *H* is the optical transformation function.

In the frequency domain the degradation process is equivalent to that of convolving the image with an optical transformation function. Similarly, the restoration can be referred to as deconvolution:

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)} \tag{4.4.2}$$

This deconvolution process can be directly applied to solving the practical problems. As an example, the **Wiener filter** is particularly used here for further discussion. The Fourier transform of the restoration is

$$\begin{aligned}
&\hat{F}(u,v) \\
&= \left[\frac{H^*(u,v)S_1(u,v)}{S_1(u,v)\,|H(u,v)|^2 + S_2(u,v)}\right]G(u,v) \\
&= \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_2(u,v)/S_1(u,v)}\right]G(u,v)
\end{aligned} \tag{4.4.3}$$

where $H(u,v)$ is degradation function, $H^*(u,v)$ is the complex conjúgate of $H(u,v)$, $|H(u,v)|^2 = H^*(u,v)\,H(u,v)$, $S_1(u,v) = |N(u,v)|^2$ that is the power spectrum of the noise, and $S_2(u,v) = |F(u,v)|^2$ that is the power spectrum of the clean image. The above equation indicates that the power spectra of the restored image depends on the power spectra of the transformation function and the ratio of the power spectrum of the noise and the clean image.

The second example is **notch filter**. This filter has a similar concept to that of bandpass and Butterworth filters. It rejects or passes frequencies in a pre-defined areas around the central frequency. Now, the rejection case is investigated as an example. Let the centre and its symmetry be (u_0, v_0) and $(-u_0, -v_0)$ respectively. If the noth reject filter is applied to a radius D_c, then the transfer function can be expressed as

$$H(u,v) = \begin{cases} 0 \\ 1 \end{cases} \tag{4.4.4}$$

where $D_c \geq D_1$ or $D_c \leq D_2$ leads to the upper case with

$$D_1(u,v) = [(u - M/2 - u_0)^2 + (v - N/2 - v_0)^2]^{\frac{1}{2}} \tag{4.4.5}$$

and

$$D_2(u,v) = [(u - M/2 + u_0)^2 + (v - N/2 + v_0)^2]^{\frac{1}{2}} \tag{4.4.6}$$

The filter will look at an area with the shifted centre to ($M/2$, $N/2$). Similarly, the transfer function of a Butterworth notch reject filter of order 2 is derived as follows:

$$H(u,v) = \frac{1}{1 + [\frac{D_c^2}{D_1(u,v)D_2(u,v)}]^2} \tag{4.4.7}$$

A Gaussian notch reject filter has the transfer function

$$H(u,v) = 1 - e^{\frac{1}{2}[\frac{D_1(u,v)D_2(u,v)}{D_c^2}]} \tag{4.4.8}$$

Another typical example is **constrained least squares filter**. The Wiener filter has an evident drawback: It is desirable to know the power spectra of the clean image and noise. In spite of the existence of an approximation of the power spectra, S_1 and S_2 may not be constant and stable. Therefore, it is desirable to investigate the minmisation of the second derivative of the image for satisfaction of smoothness:

$$\min\left\{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\nabla^2 f(x,y)]^2\right\} \tag{4.4.9}$$

Subject to the following constraint:

$$\| g - H\hat{f} \|^2 = \| n \|^2 \tag{4.4.10}$$

where $||\sim||^2$ is the Euclidean vector norm, and ∇^2 is the Laplacian operator. The frequency domain solution to this minimization problem is given by the following equation:

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2} \right] G(u,v) \tag{4.4.11}$$

where γ is a parameter that can be tuned to satisfy the constraint equation. $P(u,v)$ is the Fourier transform of the function:

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \tag{4.4.12}$$

One of the optimisation techniques for computing γ is to iterate the following procedure:

Let the residual of the difference ($g - H\hat{f}$) be r_s. We use a function of γ to describe r_s

$$J(\gamma) = r_s^T r_s = \| r_s \|^2 \tag{4.4.13}$$

Let

$$\| r_s \|^2 = \| n \|^2 \pm c \tag{4.4.14}$$

where *c* is an variable that controls the estimate accuracy. Once *c* is defined, an optimal γ can be obtained by continuous adjusting in order to satisfy Equations 4.4.6 and 4.4.9.

To compute $||r_s||^2$, one can look at its Fourier transform:

$$\| R_s \|^2 = G(u,v) - H(u,v)\hat{F}(u,v) \tag{4.4.15}$$

Since

$$\| r_s \|^2 = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} r^2(x,y) \tag{4.4.16}$$

Therefore, the variance of the noise over the entire image can be expressed as follows:

$$\sigma_n^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (n(x,y) - m_n)^2 \tag{4.4.17}$$

Where

$$m_n = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} n(x,y) \tag{4.4.18}$$

Eventually, the following equations stand:

$$\| n \|^2 = MN(\sigma_n^2 + m_n^2) \tag{4.4.19}$$

This equation shows that image restoration can be performed if the variance and mean of the noise buried in the image are known beforehand.

Figure 45 illustrates the performance of the contrainted least sqaures filter in image restoration, where the overall noisy images have been restored well.

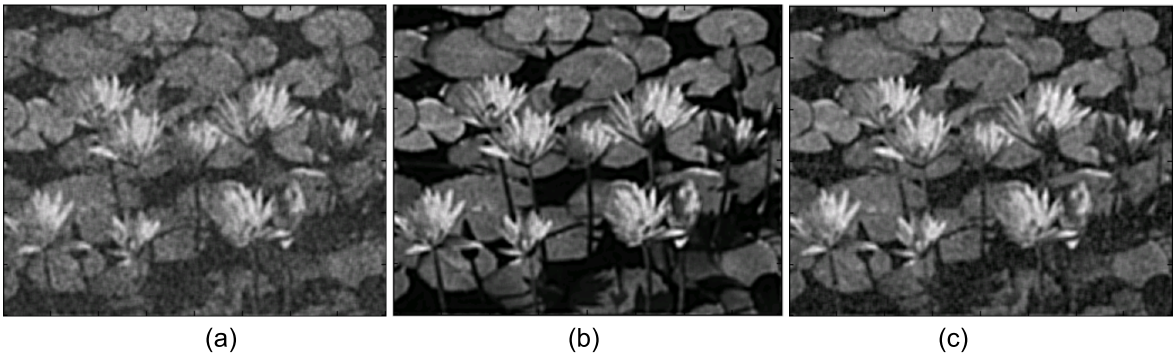


Figure 45 Examples of image restoration using the least squares filter.

4.5 Motion blur and image restoration

Motion blur is due to the relevant motion between the image sensor and the objects to be acquired during the shot period. Normally, this happens in the case where any motion is longer than the period of exposure constrained by the shutter speed. If this ocurres, the objects moving with respect to the sensor (i.e. video camera) will look blurred or smeared along the direction of the motion.

Motion blur can lead to special effects in computer graphics and visualisation. These effects sometimes are valuable. For example, in sports motion blur is used to illustrate the speed, where a slow Suter speed is normally taken. **Figure 46** clearly shows motion blur in the areas of the running path and lower bodies. In the meantime, motion Blur has been Commonly used in the animation so as to create a synthetic scene for visualisation. One of the examples is shown in **Figure 47**.

Figure 46 Motion blur in the racing scenario ([6]).

Figure 47 Motion blur in the train track scenario ([7]).

However, motion blur also causes certain side-effects. One of these effects is the nuclear details in the blurred image areas. These areas need to be sharpened so that further process can be carried out, e.g. 3-dimensional construction that requires very accurate correspondences across the image frames.

Before the blurred images can be deblurred, it will be very helpful to generate an optimal scheme if one understands how these images have been blurred. **Figure 48** denotes a synthetic motion blurred image. The strategy used to create this image follows:

- Move image pixels along a certain angle, I_a;
- Multiply the clean image I with the changed image I_a in frequency domain, I_m;
- Inverse Fourier transform of I_m.

Figure 48 Illustration of a synthetic blurred image.

The methods used to deblur the blurred images can be those that have been introduced in the previous sections, e.g. mean, Wiener and constraint least squares filters. **Figure 49** illustrates the deblurred image of **Figure 48** using the Wiener filter.

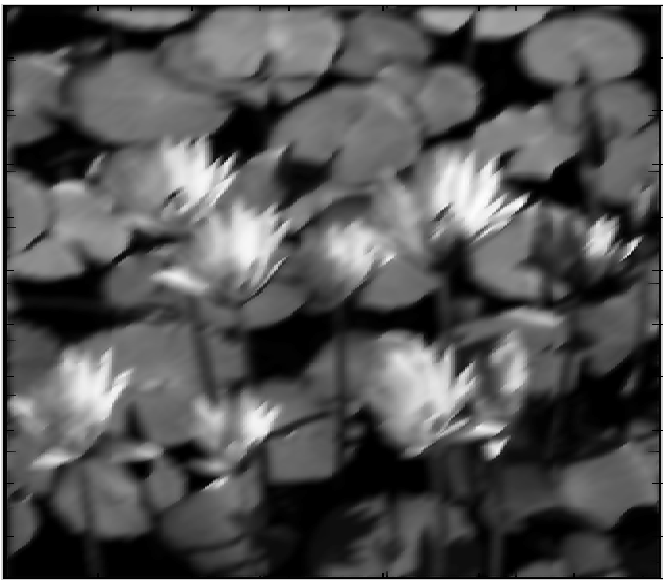


Figure 49 Illustration of the deblurred image using the Wiener filter.

4.6 Geometric transformation

The restoration techniques introduced in the previous sections are normally used to handle the images that noise signals are additive to. In this section, the case of changing the spatial relationship of the image points will be discussed. There are mainly two groups of geometric transformation: (1) Spatial transformation where the image pixels are re-arranged, and (2) image interpolation/extrapolation where a part of the images is filled or taken away.

First of all, spatial transformation is introduced. Assume an image pixel has coordinates (x, y). Then its gemetric distortion will have different coordinates as (x', y'). The relationship between the changed coordinates is

$$\begin{cases} x' = T_1(x, y) \\ y' = T_2(x, y) \end{cases} \tag{4.6.1}$$

Where T_1 and T_2 are two spatial transformation functions that generate the geometric changes. For example, a commonly used geomtric distortion is described in a pair of bilinear equations:

$$\begin{cases} T_1(x, y) = c_1 x + c_2 y + c_3 xy + c_4 \\ T_2(x, y) = c_5 x + c_6 y + c_7 xy + c_8 \end{cases} \tag{4.6.2}$$

Secondly, image interpolation/extrapolation is discussed. Due to Eq. (4.6.2), the distorted image coordinates maybe non-integer. However, image pixels can only be integer. Therefore, some "pixels" will be interpolated or extrapolated. One of the common approaches is to use a nearest neighbor approach. This approach allows the inserted or extracted image patches to have the same intensity/colour as the closest pixels. For the simplicity purpose, **Figure 50** shows a rotated image and its recovered image according to the estimated scale and angles.

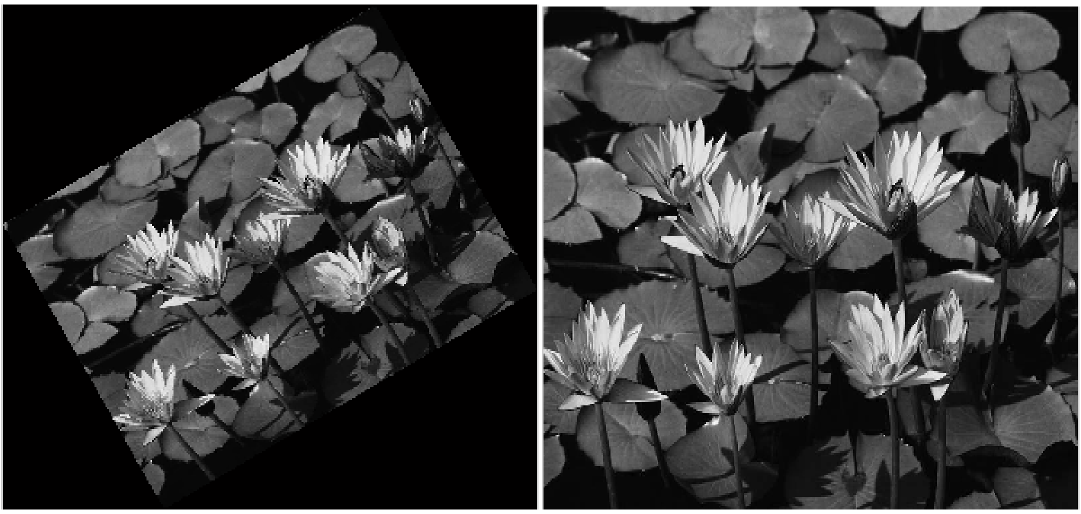


Figure 50 Illustration of the geometric distortion image (a) and its correction (b).

Summary

In this chapter, the concepts of image degradation and restoration have been introduced. The noise models are investigated before any image restoration algorithm is presented. These models consist of Gaussian, uniform, Rayleigh and salt & pepper noise. Afterwards, the corresponding techniques to restore the degraded images were also summarized. These techniques are catergorised into the spatial and frequency domains. The latter include the Wiener, notch and constrained least squares filters, while the former has examples such as the geometric mean, median, max and min filters, Wiener filter. Due to the importance and common use motion blur has been introduced in an independent section and deblur approaches are also summarized. Afterwards, geometric transformation is presented, where spatial transformation and image interpolation/extrapolation are individually described.

In general, image restoration and the developments in this field are very important in the domain of image processing. The quality of the restored images will directly affect their applications and uses in the community. Some of the possible resources of image degradation and restoration have been presented. However, many new strategies are still in their infancy stages and hence more efforts will be made in order to move forward the developments.

4.7 References

[10] http://www.sciencedaily.com/releases/2008/08/080812213810.htm, accessed on 29 September, 2009.

[11] http://www.flickr.com/photos/stevacek/299323149/, accessed on 29 September, 2009.

4.8 Problems

[15] What is image degradation/restoration?

[16] Why do we need image restoration?

[17] Please summarize the noise models that have been presented in this section.

[18] How many groups of spatial analysis are available up to date?

[19] What is the principle of motion blur? Can you apply motion blurring to the image shown below?

[20] How can we handle the motion blurred images? Hints: Provide a general algorithm for the deblurring purpose.

[21] What is geometric transformation? How can you recover a spatial distortion?

[22] Can you use a different method for deblurring **Figure 22**?

[23] Please derive Equations (4.4.7) and (4.4.8).

[24] Please try to derive Equation (4.4.11).